Calculus AB

4-3

Definition of a Definite Integral

Definition of a Definite Integral -

if f is defined on the closed interval $[a, b]$ and the limit

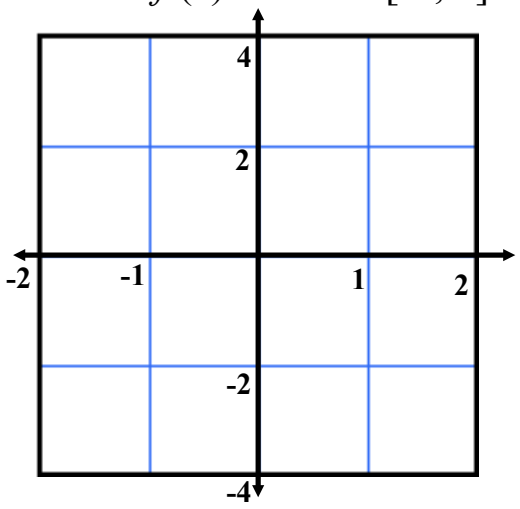
$$\lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\Delta x$$

exists, then f is integreble on $[a, b]$ and the limit is denoted by

$$\lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\,\Delta x = \int_{a}^{b} f(x)\,dx$$

The limit is called the definite integral of f from a to b, where a is the lower limit of integration, and b is the upper limit of integration.

Concept Example: Use definite integrals to find the area of $f(x) = 4x^3$ on $[-1, 1]$.

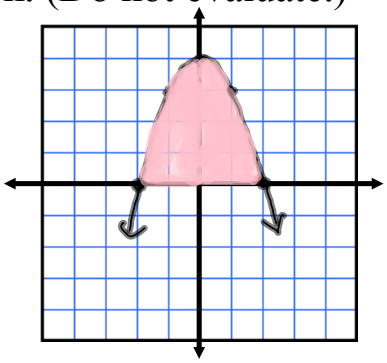


Evaluate the definite integral by the limit definition. (pg 278)

4) $\int_{-2}^{3} x\,dx =$

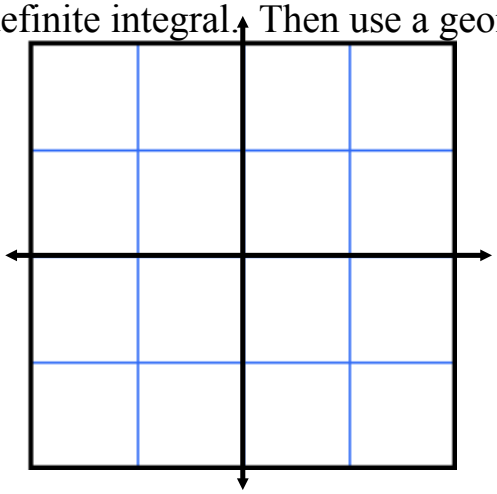
Set up a definite integral that yields the area of the region. (Do not evaluate.)

*) $f(x) = 4 - x^2$



Sketch the region whose area is given by the definite integral. Then use a geometric formula to evaluate the integral.

30) $\int_{-a}^{a} (a - |x|) \, dx$



Some Noteworthy Theorems

1) If a function f is continuous on the closed interval $[a, b]$, then f is integrable on $[a, b]$.

2) If f is defined at $x = a$, then $\int_a^a f(x)\,dx =$

3) If f is integrable on $[a, b]$, then $\int_a^b f(x)\,dx =$

4) $\int_a^b f(x)\,dx + \int_b^c f(x)\,dx =$

5) $\int_a^b k \cdot f(x)\,dx =$

6) $\int_a^b [f(x) \pm g(x)]\,dx =$

Evaluate the integral using the following values.

$$\int_2^4 x^3\,dx = 60 \qquad \int_2^4 x\,dx = 6 \qquad \int_2^4 dx = 2$$

34) $\int_4^2 x^3\,dx =$ _____

36) $\int_2^4 25\,dx =$ _____

38) $\int_2^4 (x^3 + 4)\,dx =$ _____

*) $\int_4^2 (x^3 + 4)\,dx =$ _____

Assignment:
Pg. 272
3 - 7 odd,
13 - 43 odd,
47 - 49 all.